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不确定时滞模糊系统的时滞相关鲁棒 H_∞ 控制*

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摘　要： 研究了一类带有时变时滞的不确定模糊系统时滞相关鲁棒 H_∞ 控制问题。基于模糊 Lyapunov-Krasovskii 泛函（LKF），引入多个模糊时滞自由权值矩阵，提出并证明了闭环系统新的时滞相关鲁棒 H_∞ 渐近稳定的充分条件。根据并行分布补偿算法（PDC）设计了反馈控制器，控制器可由线性矩阵不等式（LMI）求解得到。数例仿真验证了所提方法的有效性。

关键词： 模糊 Lyapunov-Krasovskii 泛函；时滞自由权值矩阵；鲁棒 H_∞ 控制；线性矩阵不等式；时滞相关
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Delay-dependent Robust H_∞ Control for Uncertain Fuzzy Systems with Delay

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Abstract: A delay-dependent robust H_∞ control problem for a class of uncertain T-S fuzzy systems with delay is presented. Based on a fuzzy Lyapunov-Krasovskii functional (LKF) and fuzzy free-weighting matrices with time-delay, a new delay-dependent robust stability criterion is proposed and proved for the close-loop fuzzy system. The feedback controller design involves solving a set of linear matrix inequalities (LMIs) via parallel distributed compensation (PDC) approach. Finally, simulation results are presented to show the effectiveness of the approach.

Key words: fuzzy Lyapunov-Krasovskii functional; free-weighting matrices with timedelay; robust H_∞ control; linear matrix inequality; delay-dependent

非线性是工业控制中普遍存在的现象，基于 T－S 模型的模糊控制是一种研究非线性系统比较成功的方法[1-8]。不确定性在实际系统中是普遍存在的，其研究有着很强的应用背景[1-2]。时滞现象也常存在于许多实际系统中，其存在会引起系统性能的下降，甚至导致系统的不稳定，因此对模糊时滞系统的研究引起了许多学者的关注[2-6]。其研究结果分为时滞无关和时滞相关[3-6]。通常时滞相关较时滞无关有小的保守性，特别在时滞较小的情况下。对模糊时滞系统进行稳定分析时，常用模型转换方法和边界不等式来估计交叉乘积项的上界[5-6]。为减少保守性，文［6，9］分别引入了自由权值矩阵和模糊自由权值矩阵方法。但上述成果多是基于公共的 Lyapunov-Krasovskii 泛函（LKF）[2,4]。为了减少公共 LKF 方法的保守性，文［6］提出了模糊 LKF 方法研究时滞系统的稳定性，但没有考虑不确定问题和 H_∞ 性能。

本文研究了一类带有时变时滞的不确定模糊系统的鲁棒 H_∞ 控制问题。定义一个模糊 LKF，并且在推导过程中引入多个包含时滞项的模糊自由权值矩阵。根据并行分布补偿算法（PDC），得到了闭环系统时滞相关的鲁棒渐近稳定条件。

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1 系统描述

由 T－S 模型描述的带有时变时滞的不确定非线性系统，它的第 i 条规则可描述如下

R^i if $\xi_1(t)$ is F_1^i

and... and $\xi_v(t)$ is F_v^i

$$\text{then}\quad \dot{x}(t) = (A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t-d) + B_i u(t) + B_{wi}w(t) \tag{1}$$

$$z(t) = (C_i + \Delta C_i)x(t) + D_i u(t)$$

$$x(t) = \varphi(t), t \in [-\tau, 0]$$

其中，$i \in S = \{1,2,\ldots,r\}$，$r$ 是模糊规则的数目。$F_j^i, j = 1,2\ldots v$ 是模糊集合，$\xi_j(t)$ 是前提变量。$x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, z(t) \in \mathbb{R}^q$ 分别是状态变量，控制输入和输出，$w(t) \in \mathbb{R}^p$ 是扰动输入且 $w(t) \in L_2[0,\infty)$。$A_i, A_{di}, B_i, B_{wi}, C_i, D_i$ 是已知的系统矩阵，$\Delta A_i, \Delta A_{di}, \Delta C_i$ 是时变不确定矩阵，有界且满足

$$\begin{bmatrix} \Delta A_i & \Delta A_{di} \\ \Delta C_i & 0 \end{bmatrix} = \begin{bmatrix} H_{1i} \\ H_{2i} \end{bmatrix} F_i(t) \begin{bmatrix} E_{ai} & E_{adi} \\ E_{ci} & 0 \end{bmatrix}$$

$E_{ai}, E_{adi}, E_{ci}, H_{1i}, H_{2i}$ 是常数矩阵，$F_i(t)$ 是未知时变矩阵，其元素 Lebesgue 可测且对任意的 t 满足 $F_i^T(t)F_i(t) \leqslant I$。$\varphi(t)$ 是系统的初始状态，时滞项 d 是时变可微函数且满足：$0 \leqslant d \leqslant \tau, \dot{d} \leqslant \sigma$，$\tau$ 和 σ 是常数。

通过单点模糊化，乘积推理和中心平均反模糊化方法，模糊系统的总体模型为：

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(\xi(t))[(A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{adi})x(t-d) + B_i u(t) + B_{wi}w(t)] \\ z(t) = \sum_{i=1}^{r} h_i(\xi(t))(C_i + \Delta C_i)x(t) \end{cases} \tag{2}$$

其中 $h_i(\xi(t)) = \omega_i(\xi(t))/\sum_{i=1}^{r}\omega_i(\xi(t)), \omega_i(\xi(t)) = \prod_{j=1}^{v}\mu_{ij}(\xi(t))$。$\mu_{ij}(\xi(t))$ 是 $\xi_j(t)$ 在 F_j^i 中的隶属度函数，$\omega_i(\xi(t)) \geqslant 0, \sum_{i=1}^{r}\omega_i(\xi(t)) > 0$。由 $h_i(\xi(t))$ 的定义可知：$h_i(\xi(t)) \geqslant 0$，$\sum_{i=1}^{r} h_i(\xi(t)) = 1$。以下在不引起混淆的情况下记 $x(t-d)$ 为 $x_d(t)$。

根据 PDC 设计控制器，第 i 个子系统的控制律为：

R^i if $\xi_1(t)$ is F_1^i

and... and $\xi_v(t)$ is F_v^i (3)

then $u(t) = K_i x(t)$

整个系统的控制律可表示为：

$$u(t) = \sum_{i=1}^{r} h_i(\xi(t))K_i x(t) \tag{4}$$

把（4）代入（2）中，闭环系统可表示为：

$$\begin{cases} \dot{x}(t) = \sum_{i,j=1}^{r} h_i(\xi(t))h_j(\xi(t))[(A_i + B_iK_j + \Delta A_i)x(t) + (A_{di} + \Delta A_{adi})x_d(t) + B_{wi}w(t)] \\ z(t) = \sum_{i,j=1}^{r} h_i(\xi(t))h_j(\xi(t))(C_i + D_iK_j + \Delta C_i)x(t) \end{cases} \tag{5}$$

定义 1　对于给定的常数 $\rho > 0$，若系统（5）满足：①当 $w(t) \equiv 0$ 时，系统（5）是渐近稳定的；②在零初始条件下 $\varphi(t) = 0, t \in [-\tau, 0]$，对任意非零 $w(t) \neq 0$，满足 $\|z\|_2 < \rho\|w\|_2$。则称系统（5）在 H_∞ 性能指标 ρ 下鲁棒渐近稳定。

以下给出在证明中要用到的引理：

引理 1[10]　设 M, N 和 $F(t)$ 是维数适合的实矩阵且满足 $F^T(t)F(t) \leqslant I$，则对于 $\varepsilon > 0$ 有如下不等式成立：$M^TF(t)N + N^TF^T(t)M \leqslant \varepsilon M^TM + \varepsilon^{-1}N^TN$。

引理 2[8]　设 A, D, E, F 是合适维数的实数矩阵，且 $F^T(t)F(t) \leqslant I$，则有矩阵 $P > 0$ 对于标量 $\varepsilon > 0$ 满足 $\varepsilon I - D^TPD > 0$ 时，有如下不等式成立

$$(A + DFE)^TP(A + DFE) \leqslant A^TPA + A^TPD(\varepsilon I - D^TPD)^{-1}D^TPA + \varepsilon E^TE$$

2 主要结果

2.1 稳定性分析

引入带有时滞的模糊自由权值矩阵

$$\bar{X} = \begin{bmatrix} X \\ X_4 \end{bmatrix} = \begin{bmatrix} X_1(t) \\ X_2(t-d) \\ X_3(t) \\ X_4(t) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{r} h_i(\xi(t))X_{1i} \\ \sum_{l=1}^{r} h_l(\xi(t-d))X_{2i} \\ \sum_{i=1}^{r} h_i(\xi(t))X_{3i} \\ \sum_{i=1}^{r} h_i(\xi(t))X_{4i} \end{bmatrix}$$

$$\bar{Y} = [Y\ Y_4]^T = [Y_1\ Y_2\ Y_3\ Y_4]^T \tag{6}$$

其中 $X_{ki}, Y_k, i \in S, k = 1,2,3,4$ 是待定的合适维数的常数矩阵。以下分别记 $X_i(t)$ 为 $X_i, i = 1,3,4$，记 $X_2(t-d)$ 为 X_2。

定理 1　对于给定的常数 $\rho > 0, \tau > 0$ 和 $\sigma >$

0，如果对于正常数 $\varepsilon_m, m = 1,2,\ldots,5$ 存在正定对称矩阵 P, Q_i, R_i, X_{ki} 和 $Y_k, k = 1,2,3,4, i \in S$ 满足矩阵不等式（7）和（8），则系统（5）在 H_∞ 性能指标 ρ 下是鲁棒渐近稳定的。

$$\begin{bmatrix} \bar{\Phi}_{ii,l} & \tau \bar{X}_i \\ * & -\tau R_m \end{bmatrix} < 0, \quad i,l,m \in S \tag{7}$$

$$\begin{bmatrix} \bar{\Phi}_{ij,l} + \bar{\Phi}_{ji,l} & \tau \bar{X}_i & \tau \bar{X}_j \\ * & -\tau R_m & 0 \\ * & * & -\tau R_m \end{bmatrix} < 0, \quad i < j, l, m \in S \tag{8}$$

其中 $\bar{\Phi}_{ij,l} = \begin{bmatrix} \bar{\varphi}_{11,ij} & \varphi_{12,ij} & \varphi_{13,ij} & \varphi_{14,ij} \\ * & \bar{\varphi}_{22,i,l} & \varphi_{23,i} & \varphi_{24,i} \\ * & * & \varphi_{33,i} & \varphi_{34,i} \\ * & * & * & \varphi_{44,i} \end{bmatrix}$

$$\Phi_{ij,l} = \begin{bmatrix} \varphi_{11,ij} & \varphi_{12,ij} & \varphi_{13,ij} \\ * & \varphi_{22,i,l} & \varphi_{23,i} \\ * & * & \varphi_{33,i} \end{bmatrix}$$

$$\varphi_{11,ij} = Q_i + X_{1i} + X_{1i}^{\mathrm T} + Y_1(A_i + B_iK_j) + (A + B_iK_j)^{\mathrm T} Y_1^{\mathrm T} + 2\varepsilon_1 Y_1 H_{1i} H_{1i}^{\mathrm T} Y_1^{\mathrm T} + a E_{ai}^{\mathrm T} E_{ai}$$

$$\varphi_{ij} = (C_i + D_iK_j)^{\mathrm T}(C_i + D_iK_j) + (C_i + D_iK_j)^{\mathrm T} H_{2i}(\varepsilon_5 I - H_{2i}^{\mathrm T} H_{2i})^{-1} H_{2i}^{\mathrm T}(C_i + D_iK_j) + \varepsilon_5 E_{ci}^{\mathrm T} E_{ci}$$

$$\bar{\varphi}_{11,ij} = \varphi_{11,ij} + (\bar{a} - a) E_{ai}^{\mathrm T} E_{ai} + \varphi_{ij}$$

$$\varphi_{12,ij} = -X_{1i} + X_{2i}^{\mathrm T} + Y_1 A_{di} + (A_i + B_iK_j)^{\mathrm T} Y_2^{\mathrm T}$$

$$\varphi_{13,ij} = P + X_{3i}^{\mathrm T} + (A_i + B_iK_j)^{\mathrm T} Y_{3i}^{\mathrm T} - Y_1$$

$$\varphi_{14,ij} = X_{4i}^{\mathrm T} + Y_1 B_{wi} + (A_i + B_iK_j)^{\mathrm T} Y_4^{\mathrm T}$$

$$\varphi_{22,i,l} = -(1-\sigma) Q_1 - X_{2i} - X_{2i}^{\mathrm T} + Y_2 A_{di} + A_{di}^{\mathrm T} Y_2^{\mathrm T} + 2\varepsilon_2 Y_2 H_{1i} H_{1i}^{\mathrm T} Y_2^{\mathrm T} + a E_{adi}^{\mathrm T} E_{adi}$$

$$\bar{\varphi}_{22,i,l} = \varphi_{22,i,l} + (\bar{a} - a) E_{adi}^{\mathrm T} E_{adi}$$

$$\varphi_{23,i} = -X_{3i}^{\mathrm T} + A_{di}^{\mathrm T} Y_3^{\mathrm T} - Y_2$$

$$\varphi_{24,i} = -X_{4i}^{\mathrm T} + Y_2 B_{wi} + A_{di}^{\mathrm T} Y_4^{\mathrm T}$$

$$\varphi_{33,i} = \tau R_i - Y_3 - Y_3^{\mathrm T} + 2\varepsilon_3 Y_3 H_{1i} H_{1i}^{\mathrm T} Y_3^{\mathrm T}$$

$$\varphi_{34,i} = Y_3 B_{wi} - Y_4^{\mathrm T}$$

$$\varphi_{44,i} = Y_4 B_{wi} + B_{wi}^{\mathrm T} Y_4^{\mathrm T} + 2\varepsilon_4 Y_4 H_{1i} H_{1i}^{\mathrm T} Y_4^{\mathrm T} - \rho^2 I$$

$$a = \varepsilon_1^{-1} + \varepsilon_2^{-1} + \varepsilon_3^{-1}, \bar{a} = a + \varepsilon_4^{-1}$$

证明 选取模糊 Lyapunov-Krasovkii 泛函：

$$V(x(t)) = x^{\mathrm T}(t) P x(t) + \int_{t-d}^{t} x^{\mathrm T}(s) Q(s) x(s) \mathrm{d}s + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm T}(s) R(s) \dot{x}(s) \mathrm{d}s\mathrm{d}\theta \tag{9}$$

其中 $Q(t) = \sum_{i=1}^{r} h_i(\xi(t)) Q_i$，$R(t) = \sum_{i=1}^{r} h_i(\xi(t)) R_i$。这里 P 没有用 $\sum_{i=1}^{r} h_i(\xi(t)) P_i$ 来代替，是为了避免确定 $|h_i(\xi(t))|$ 的上界而带来的保守性[7]。

首先考虑 $w(t) \equiv 0$，时闭环系统的渐近稳定性。当 $w(t) \equiv 0$，闭环系统（5）可表示为：

$$\dot{x}(t) = \sum_{i,j=1}^{r} h_i(\xi(t)) h_j(\xi(t)) [(A_i + B_iK_j + \Delta A_i) x(t) + (A_{di} + \Delta A_{adi}) x_d(t)] \tag{10}$$

沿着系统（10）的轨线，对 $V(x(t))$ 求导，由 Leibniz-Newton 公式，可得到：

$$\begin{aligned}\dot{V}(x(t)) \leqslant & \sum_{i,l,m=1}^{r} h_i(\xi(t)) h_l(\xi(t-d)) \times h_m(\xi(s)) [2x^{\mathrm T}(t) P \dot{x}(t) + x^{\mathrm T}(t) Q_i x(t) - (1-\sigma) x_d^{\mathrm T}(t) Q_l x_d(t) + \tau \dot{x}^{\mathrm T}(t) R_i \dot{x}(t) - \\ & \int_{t-d}^{t} \dot{x}^{\mathrm T}(s) R_m \dot{x}(s) \mathrm{d}s] + 2\sum_{i=1}^{r} h_i(\xi(t)) \times [x^{\mathrm T}(t) X_{1i} + x_d^{\mathrm T}(t) X_{2i} + \dot{x}^{\mathrm T}(t) X_{3i}][x(t) - x_d(t) - \int_{t-d}^{t} \dot{x}(s) \mathrm{d}s] + 2[x^{\mathrm T}(t) Y_1 + \\ & x_d^{\mathrm T}(t) Y_2 + \dot{x}^{\mathrm T}(t) Y_3] \sum_{i,j=1}^{r} h_i(\xi(t)) h_j(\xi(t)) [(A_i + B_iK_j + \Delta A_i) x(t) + (A_{di} + \Delta A_{adi}) x_d(t) - \dot{x}(t)]\end{aligned} \tag{11}$$

由引理 1 知

$$\begin{aligned}
2x^{\mathrm T}(t) Y_1 \Delta A_i x(t) &\leqslant \varepsilon_1 x^{\mathrm T}(t) Y_1 H_{1i} H_{1i}^{\mathrm T} Y_1^{\mathrm T} x(t) + \varepsilon_1^{-1} x^{\mathrm T}(t) E_{ai}^{\mathrm T} E_{ai} x(t) \\
2x^{\mathrm T}(t) Y_1 \Delta A_{di} x_d(t) &\leqslant \varepsilon_1 x^{\mathrm T}(t) Y_1 H_{1i} H_{1i}^{\mathrm T} Y_1^{\mathrm T} x(t) + \varepsilon_1^{-1} x_d^{\mathrm T}(t) E_{adi}^{\mathrm T} E_{adi} x_d(t) \\
2x_d^{\mathrm T}(t) Y_2 \Delta A_i x(t) &\leqslant \varepsilon_2 x_d^{\mathrm T}(t) Y_2 H_{1i} H_{1i}^{\mathrm T} Y_2^{\mathrm T} x_d(t) + \varepsilon_2^{-1} x^{\mathrm T}(t) E_{ai}^{\mathrm T} E_{ai} x(t) \\
2x_d^{\mathrm T}(t) Y_2 \Delta A_{di} x_d(t) &\leqslant \varepsilon_2 x_d^{\mathrm T}(t) Y_2 H_{1i} H_{1i}^{\mathrm T} Y_2^{\mathrm T} x_d(t) + \varepsilon_2^{-1} x_d^{\mathrm T}(t) E_{adi}^{\mathrm T} E_{adi} x_d(t) \\
2\dot{x}^{\mathrm T}(t) Y_3 \Delta A_i x(t) &\leqslant \varepsilon_3 \dot{x}^{\mathrm T}(t) Y_3 H_{1i} H_{1i}^{\mathrm T} Y_3^{\mathrm T} x\dot{x}(t) + \varepsilon_3^{-1} x^{\mathrm T}(t) E_{ai}^{\mathrm T} E_{ai} x(t) \\
2\dot{x}^{\mathrm T}(t) Y_3 \Delta A_{di} x_d(t) &\leqslant \varepsilon_3 \dot{x}^{\mathrm T}(t) Y_3 H_{1i} H_{1i}^{\mathrm T} Y_3^{\mathrm T} \dot{x}(t) + \varepsilon_3^{-1} x_d^{\mathrm T}(t) E_{adi}^{\mathrm T} E_{adi} x_d(t)
\end{aligned} \tag{12}$$

记 $\bar{\eta}(t) = [\eta^{\mathrm T}(t) w^{\mathrm T}(t)]^{\mathrm T} = [x^{\mathrm T}(t) x_d^{\mathrm T}(t) \dot{x}^{\mathrm T}(t) w^{\mathrm T}(t)]^{\mathrm T}$，把（12）代入（11）得：

$$\begin{aligned}\dot{V}(x(t)) \leqslant & \sum_{i,j,l,m=1}^{r} h_i(\xi(t)) h_j(\xi(t)) h_l(\xi(t-d)) h_m(\xi(s)) \eta^{\mathrm T}(t) (\Phi_{ij,l} + \tau X_i R_m^{-1} X_i^{\mathrm T}) \times \eta(t) - \\ & \int_{t-d}^{t} (\eta^{\mathrm T}(t) X + \dot{x}^{\mathrm T}(s) R(s)) R^{-1}(s) (\eta^{\mathrm T}(t) X + \dot{x}^{\mathrm T}(s) R(s))^{\mathrm T} \mathrm{d}s =\end{aligned}$$

$$\sum_{i,l,m=1}^{r} h_i^2(\xi(t))h_l(\xi(t-d))h_m(\xi(s))\eta^{T}(t)(\Phi_{ii,l}+\tau X_i R_m^{-1}X_i^{T})\eta(t)+\sum_{1=i<j;l,m=1}^{r} h_i(\xi(t))h_j(\xi(t))h_l(\xi(t-d))h_m(\xi(s))\eta^{T}(t)\times(\Phi_{ij,l}+\Phi_{ji,l}+\tau X_i R_m^{-1}X_i^{T}+\tau X_j R_m^{-1}X_j^{T})\eta(t)-\int_{t-d}^{t}(\eta^{T}(t)X+\dot{x}^{T}(s)R(s))R^{-1}(s)\times(\eta^{T}(t)X+\dot{x}^{T}(s)R(s))^{T}ds \quad (13)$$

考虑定理 1 中的 (7) - (8) 式分别等价于

$$\begin{bmatrix} \bar{\varphi}_{11,ii} & \varphi_{12,ii} & \varphi_{13,ii} & \tau X_{1i} \\ * & \bar{\varphi}_{22,i,l} & \varphi_{23,i} & \tau X_{2i} \\ * & * & \varphi_{33,i} & \tau X_{3i} \\ * & * & * & -\tau R_m \end{bmatrix} < 0,\quad i,l,m\in S \quad (14)$$

$$\begin{bmatrix} \bar{\varphi}_{11,ij}+\bar{\varphi}_{11,ji} & \varphi_{12,ij}+\varphi_{12,ji} & \varphi_{13,ij}+\varphi_{13,ji} & \tau X_{1i} & \tau X_{1j} \\ * & \bar{\varphi}_{22,i,l}+\bar{\varphi}_{22,j,l} & \varphi_{23,i}+\varphi_{23,j} & \tau X_{2i} & \tau X_{2j} \\ * & * & \varphi_{33,i}+\varphi_{33,j} & \tau X_{3i} & \tau X_{3j} \\ * & * & * & -\tau R_m & 0 \\ * & * & * & * & -\tau R_m \end{bmatrix} < 0,\quad i<j,l,m\in S \quad (15)$$

再次使用 Schur 补定理，由 (15) - (16) 可分别得到：$\Phi_{ij,l}+\Phi_{ji,l}+\tau X_i R_m^{-1}X_i^{T}+\tau X_j R_m^{-1}X_j^{T}<0$ 和 $\Phi_{ii,l}+\tau X_i R_m^{-1}X_i^{T}<0$。考虑 $R_m>0$，可知 (13) 式最后一项是非正项，可得 $\dot{V}(x(t))<0$，则闭环系统 (10) 是渐近稳定的。

以下考虑零初始条件 $\varphi(t)=0, t\in[-\tau,0]$，$w(t)\neq 0$ 时的闭环系统 (5) 的 H_∞ 性能。沿着系统 (5) 的轨线对 $V(x(t))$ 求导，并引入公式 Leibniz-Newton 和系统方程 (5)，可得：

$$\dot{V}(x(t))\leqslant\sum_{i,l,m=1}^{r}h_i(\xi(t))h_l(\xi(t-d))h_m(\xi(s))[2x^{T}(t)P\dot{x}(t)+x^{T}(t)Q_i x(t)-(1-\sigma)x_d^{T}(t)Q_l\times x_d(t)+\tau\dot{x}^{T}(t)R_i\dot{x}(t)-\int_{t-d}^{t}\dot{x}^{T}(s)R_m\dot{x}(s)ds]+2\sum_{i=1}^{r}h_i(\xi(t))[x^{T}(t)X_{1i}+x_d^{T}(t)X_{2i}+\dot{x}^{T}(t)\times X_{3i}+w^{T}(t)X_{4i}][x(t)-x_d(t)-\int_{t-d}^{t}\dot{x}(s)ds)]+2[x^{T}(t)Y_1+x_d^{T}(t)Y_2+\dot{x}^{T}(t)Y_3+w^{T}(t)Y_4]\times\sum_{i,j=1}^{r}h_i(\xi(t))h_j(\xi(t))[(A_i+B_iK_j+\Delta A_i)x(t)+(A_{di}+\Delta A_{adi})x_d(t)+B_{wi}w(t)-\dot{x}(t)] \quad (16)$$

同样由引理 1 可得：

$$2w^{T}(t)Y_4\Delta A_i x(t)\leqslant\varepsilon_4 w^{T}(t)Y_4H_{1i}H_{1i}^{T}Y_4^{T}w(t)+\varepsilon_4^{-1}x^{T}(t)E_{ai}^{T}E_{ai}x(t)$$

$$2w^{T}(t)Y_4\Delta A_{di}x_d(t)\leqslant\varepsilon_4 w^{T}(t)Y_4H_{1i}H_{1i}^{T}Y_4^{T}w(t)+\varepsilon_4^{-1}x_d^{T}(t)E_{adi}^{T}E_{adi}x_d(t) \quad (17)$$

由引理 2 可知 (14) 式可修正为：

$$z^{T}(t)z(t)-\rho^2 w^{T}(t)w(t)\leqslant\sum_{i,j=1}^{r}h_i(\xi(t))h_j(\xi(t))x^{T}(t)\varphi_{ij}x(t)-\rho^2 w^{T}(t)w(t) \quad (18)$$

把 (17) 式代入 (16) 式，并结合 (18) 式可以得到：$z^{T}(t)z(t)-\rho^2 w^{T}(t)w(t)+\dot{V}(x(t))\leqslant$

$$\sum_{i,j,l,m=1}^{r}h_i(\xi(t))h_j(\xi(t))h_l(\xi(t-d))h_m(\xi(s))\bar{\eta}^{T}(t)(\bar{\Phi}_{ij,l}+\tau\bar{X}_iR_m^{-1}\bar{X}_i^{T})\bar{\eta}(t)-\int_{t-d}^{t}(\bar{\eta}^{T}(t)\bar{X}+\dot{x}^{T}(s)R(s))R^{-1}(s)(\bar{\eta}^{T}(t)\bar{X}+\dot{x}^{T}(s)R(s))^{T}ds$$

$$=\sum_{i,l,m=1}^{r}h_i^2(\xi(t))h_l(\xi(t-d))h_m(\xi(s))\bar{\eta}^{T}(t)(\bar{\Phi}_{ii,l}+\tau\bar{X}_iR_m^{-1}\bar{X}_i^{T})\bar{\eta}(t)+\sum_{1=i<j;l,m=1}^{r}h_i(\xi(t))\times h_j(\xi(t))h_l(\xi(t-d)h_m(\xi(s))\bar{\eta}^{T}(t)(\bar{\Phi}_{ij,l}+\bar{\Phi}_{ji,l}+\tau\bar{X}_iR_m^{-1}\bar{X}_i^{T}+\tau\bar{X}_jR_m^{-1}\bar{X}_j^{T})\bar{\eta}(t)-\int_{t-d}^{t}(\bar{\eta}^{T}(t)\bar{X}+\dot{x}^{T}(s)R(s))R^{-1}(s)(\bar{\eta}^{T}(t)\bar{X}+\dot{x}^{T}(s)R(s))^{T}ds \quad (19)$$

使用 Schur 补定理，由定理 1 可知：

$$\dot{V}(x(t))+z^{T}(t)z(t)-\rho^2 w^{T}(t)w(t)<0 \quad (20)$$

积分后可得：

$$\int_0^t[z^{T}(t)z(t)-\rho^2 w^{T}(t)w(t)]dt+V(t)-V(0)<0 \quad (21)$$

对上式取极限，$\|z\|^2<\rho^2\|w\|^2$，系统 (5) 在 H_∞ 性能指标 ρ 下是鲁棒渐近稳定的。

2.2　控制器的设计

考虑定理 1 中的矩阵不等式中含有耦合项，为求解控制器，这里设 Y_1,Y_2,Y_3,Y_4 是非奇异矩阵且 $Y_k^{-T}=\lambda_k Z, Z=P^{-1}, k=1,2,3,4, \lambda_k>0, k=1,2,3,4$ 是常数。记：$M_i=K_iZ$，$\bar{Q}(t)=\sum_{i=1}^{r}h_i(\xi(t))\bar{Q}_i=\sum_{i=1}^{r}h_i(\xi(t))Y_1^{-1}Q_iY_1^{-T}$，$\bar{R}(t)=\sum_{i=1}^{r}h_i(\xi(t))\bar{R}_i=\sum_{i=1}^{r}h_i(\xi(t))Y_3^{-1}R_iY_3^{-T}$，$\bar{X}_k(t)=Y_k^{-1}X_kY_k^{-T}$，$k=1,3,4$，$\bar{X}_2(t-d)=Y_2^{-1}X_2Y_2^{-T}$，$i\in S$。同样用 $\bar{X}_1,\bar{X}_2,\bar{X}_3,\bar{X}_4$ 来表示 $\bar{X}_1(t),\bar{X}_2(t-d(t)),\bar{X}_3(t),\bar{X}_4(t)$。分别定义 $\Theta=\mathrm{diag}\{Y_1^{-1},Y_2^{-1},Y_3^{-1},Y_4^{-1},Y_3^{-1}\}$ 和 $\bar{\Theta}=\mathrm{diag}\{Y_1^{-1},Y_2^{-1},Y_3^{-1},Y_4^{-1},Y_3^{-1},Y_3^{-1}\}$，对 (14) - (15) 式分别左右乘 Θ,Θ^{T} 和 $\bar{\Theta},\bar{\Theta}^{T}$，并由 Schur 补定理可得到：

$$\begin{bmatrix} \Phi_{ii,lm}^{(1)} & \Phi_{ii}^{(2)} \\ * & \Phi_{ii}^{(3)} \end{bmatrix} < 0, \quad i,l,m \in S \qquad (22)$$

$$\begin{bmatrix} \Phi_{ij,lm}^{(4)} & \Phi_{ij}^{(5)} \\ * & \Phi_{ij}^{(6)} \end{bmatrix} < 0, \quad i < j,l,m \in S \qquad (23)$$

其中

$$\Phi_{ii,lm}^{(1)} = \begin{bmatrix} \varphi_{11,ii}^{(2)} & \varphi_{12,ii}^{(1)} & \varphi_{13,ii}^{(1)} & \varphi_{14,ii}^{(1)} & \tau\lambda_1^{-1}\lambda_3\bar{X}_{1i} \\ & \varphi_{22,i,l}^{(1)} & \varphi_{23,i}^{(1)} & \varphi_{24,i}^{(1)} & \tau\lambda_2^{-1}\lambda_3\bar{X}_{2i} \\ & & \varphi_{33,i}^{(1)} & \varphi_{34,i}^{(1)} & \tau\bar{X}_{3i} \\ & & & \varphi_{44,i}^{(1)} & \tau\lambda_4^{-1}\lambda_3\bar{X}_{4i} \\ & & & & -\tau\bar{R}_m \end{bmatrix}$$

$$\Phi_{ii}^{(2)} = \mathrm{diag}\{\varphi_{11,ii}^{(2)},\varphi_{22,ii}^{(2)},0,\varphi_{44}^{(2)}\}$$
$$\Phi_{ii}^{(3)} = \mathrm{diag}\{\varphi_{11,ii}^{(3)},\varphi_{22}^{(3)},0,\varphi_{44}^{(3)}\}$$
$$\Phi_{ij}^{(5)} = \mathrm{diag}\{\varphi_{11,ij}^{(5)},\varphi_{22,ij}^{(5)},0,\varphi_{44}^{(5)}\}$$
$$\Phi_{ij}^{(6)} = \mathrm{diag}\{\varphi_{11,ij}^{(6)},\varphi_{22}^{(6)},0,\varphi_{44}^{(6)}\}$$

$$\varphi_{11,ij}^{(1)} = \bar{Q}_i + \bar{X}_{1i} + \bar{X}_{1i}^{\mathrm{T}} + \lambda_1((A_iZ + B_iM_j) + (A_iZ + B_iM_j)^{\mathrm{T}}) + 2\varepsilon_1 H_{1i}H_{1i}^{\mathrm{T}}$$

$$\varphi_{12,ij}^{(1)} = -\lambda_1^{-1}\lambda_2\bar{X}_{1i} + \lambda_2^{-1}\lambda_1\bar{X}_{2i}^{\mathrm{T}} + \lambda_2 A_{di}Z + \lambda_1(A_iZ + B_iM_j)^{\mathrm{T}}$$

$$\varphi_{13,ij}^{(1)} = \lambda_1\lambda_3 Z + \lambda_3^{-1}\lambda_1\bar{X}_{3i}^{\mathrm{T}} + \lambda_1(A_iZ + B_iM_j)^{\mathrm{T}} - \lambda_3 Z$$

$$\varphi_{14,ij}^{(1)} = \lambda_4^{-1}\lambda_1\bar{X}_{4i}^{\mathrm{T}} + \lambda_4^{-1}B_{wi}Z + \lambda_1(A_iZ + B_iM_j)^{\mathrm{T}}$$

$$\Phi_{ij,lm}^{(5)} = \begin{bmatrix} \varphi_{11,ij}^{(1)}+\varphi_{11,ji}^{(1)} & \varphi_{12,ij}^{(1)}+\varphi_{12,ji}^{(1)} & \varphi_{13,ij}^{(1)}+\varphi_{13,ji}^{(1)} & \varphi_{14,ij}^{(1)}+\varphi_{14,ji}^{(1)} & \tau\lambda_1^{-1}\lambda_3\bar{X}_{1i} & \tau\lambda_1^{-1}\lambda_3\bar{X}_{1j} \\ * & \varphi_{22,i,l}^{(1)}+\varphi_{22,j,l}^{(1)} & \varphi_{23,i}^{(1)}+\varphi_{23,j}^{(1)} & \varphi_{24,i}^{(1)}+\varphi_{24,j}^{(1)} & \tau\lambda_2^{-1}\lambda_3\bar{X}_{2i} & \tau\lambda_2^{-1}\lambda_3\bar{X}_{2j} \\ * & * & \varphi_{33,i}^{(1)}+\varphi_{33,j}^{(1)} & \varphi_{34,i}^{(1)}+\varphi_{34,j}^{(1)} & \tau\bar{X}_{3i} & \tau\bar{X}_{3j} \\ * & * & * & \varphi_{44,i}^{(1)}+\varphi_{44,j}^{(1)} & \tau\lambda_4^{-1}\lambda_3\bar{X}_{4i} & \tau\lambda_4^{-1}\lambda_3\bar{X}_{4j} \\ * & * & * & * & -\tau\bar{R}_m & 0 \\ * & * & * & * & * & -\tau\bar{R}_m \end{bmatrix}$$

$$\varphi_{22,i,l}^{(1)} = -(1-\sigma)\lambda_1^{-2}\lambda_2^2\bar{Q}_l - \bar{X}_{2i} - \bar{X}_{2i}^{\mathrm{T}} + \lambda_2(A_{di}Z + ZA_{di}^{\mathrm{T}}) + 2\varepsilon_2 H_{1i}H_{1i}^{\mathrm{T}}$$
$$\varphi_{23,i}^{(1)} = -\lambda_3^{-1}\lambda_2\bar{X}_{3i}^{\mathrm{T}} + \lambda_2 ZA_{di}^{\mathrm{T}} - \lambda_3 Z$$
$$\varphi_{24,i}^{(1)} = -\lambda_4^{-1}\lambda_2\bar{X}_{4i}^{\mathrm{T}} + \lambda_4 B_{wi}Z + \lambda_2 ZA_{di}^{\mathrm{T}}$$
$$\varphi_{33,i}^{(1)} = \tau\bar{R}_i - 2\lambda_3 Z + 2\varepsilon_3 H_{1i}H_{1i}^{\mathrm{T}}$$
$$\varphi_{34,i}^{(1)} = \lambda_4 B_{wi}Z - \lambda_3 Z$$
$$\varphi_{44,i}^{(1)} = \lambda_4(B_{wi}Z + ZB_{wi}^{\mathrm{T}}) + 2\varepsilon_4 H_{1i}H_{1i}^{\mathrm{T}}$$
$$\varphi_{11,ii}^{(2)} = [\lambda_1 ZE_{ai}^{\mathrm{T}}\ \lambda_1(C_iZ + D_iM_i)^{\mathrm{T}}\ \lambda_1(C_iZ + D_iM_i)^{\mathrm{T}} H_{2i}\ \lambda_1 ZE_{ci}^{\mathrm{T}}]$$
$$\varphi_{22,ii}^{(2)} = [\lambda_2 ZE_{adi}^{\mathrm{T}}]$$
$$\varphi_{44}^{(2)} = [\rho\lambda_4 Z]$$
$$\varphi_{11,ii}^{(3)} = \mathrm{diag}\{-\bar{a}^{-1}I,\ -I, H_{2i}^{\mathrm{T}}H_{2i} - \varepsilon_5 I,\ -\varepsilon_5^{-1}I\}$$
$$\varphi_{22}^{(3)} = -\bar{a}^{-1}I$$
$$\varphi_{44}^{(3)} = -I$$
$$\varphi_{11,ij}^{(5)} = [\lambda_1 ZE_{ai}^{\mathrm{T}}\quad \lambda_1 ZE_{aj}^{\mathrm{T}}\quad \lambda_1(C_iZ + D_iM_j)^{\mathrm{T}}\quad \lambda_1(C_jZ + D_jM_i)^{\mathrm{T}}\quad \lambda_1(C_iZ + D_iM_j)^{\mathrm{T}}H_{2i}\quad \lambda_1(C_jZ + D_jM_i)^{\mathrm{T}}H_{2j}\quad \lambda_1 ZE_{ci}^{\mathrm{T}}\lambda_1 ZE_{cj}^{\mathrm{T}}]$$
$$\varphi_{22,ij}^{(5)} = [\lambda_2 ZE_{adi}^{\mathrm{T}}\quad \lambda_2 ZE_{adj}^{\mathrm{T}}]$$
$$\varphi_{44}^{(5)} = [\rho\lambda_4 Z\ \rho\lambda_4 Z]$$
$$\varphi_{11,ij}^{(6)} = \mathrm{diag}\{-\bar{a}^{-1}I,\ -\bar{a}^{-1}I,\ -I,\ -I,\ H_{2i}^{\mathrm{T}}H_{2i} - \varepsilon_5 I, H_{2j}^{\mathrm{T}}H_{2j} - \varepsilon_5 I,\ -\varepsilon_5^{-1}I,\ -\varepsilon_5^{-1}I\}$$
$$\varphi_{22}^{(6)} = \mathrm{diag}\{-\bar{a}^{-1}I,\ -\bar{a}^{-1}I\}$$
$$\varphi_{44}^{(6)} = \mathrm{diag}\{-I,\ -I\}$$

综上述分析，下面给出控制器设计方法：

定理2 对于给定的正常数ρ,τ和σ，如果对于给定的正常数$\varepsilon_m, m = 1,2,\ldots,5$及$\lambda_i > 0, i = 1,2,3,4$，系统（5）存在着正定对称矩阵$Z,\bar{Q}_i,\bar{R}_i$和矩阵$M_i,\bar{X}_{ki}, k = 1,2,3,4; i \in S$满足LMIs（22）和（23），则系统（5）在$H_\infty$性能指标$\rho$下鲁棒渐近稳定，且控制器为$K_i = M_iZ^{-1}, i \in S$。

证明 根据$K_j = M_jZ^{-1}$可知$M_j = K_jZ$，代入（22）和（23）中。由（22）-（23）和（24）-（25）′的等价性可知系统（5）在H_∞性能指标ρ下是鲁棒渐近稳定的。

3 数例分析

考虑如下一个带有时滞的不确定模糊系统：

R^i: if $x_1 \in M_1$

then $\dot{x}(t) = (A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di}) x(t-d) + B_iu(t) + B_{wi}w(t)$

$z(t) = (C_i + \Delta C_i)x(t) + D_iu(t), i = 1,2$

其中，

$$A_1 = \begin{bmatrix} -0.5 & -0.3 \\ 0.1 & 1 \end{bmatrix};\quad A_2 = \begin{bmatrix} -0.5 & 0.3 \\ -0.1 & 1 \end{bmatrix}$$

$$A_{d1} = \begin{bmatrix} -0.05 & 0.1 \\ 0 & 0 \end{bmatrix};\quad A_{d2} = \begin{bmatrix} 0.05 & -0.1 \\ 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -0.1 \\ 0 \end{bmatrix}; B_2 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}; E_{a1} = E_{a2} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix}$$

$E_{ad1} = E_{ad2} = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix}$；　$H_{11} = H_{12} = \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix}$

$B_{w1} = B_{w2} = -0.5$；　$C_1 = C_2 = [-0.6 \quad 0]$

$D_1 = D_2 = 0.1$；　$E_{c1} = E_{c2} = [-0.1 \quad 0]$

$H_{21} = H_{22} = 0.3$

选取隶属度函数 $\mu_{M_1}(x_1) = \dfrac{1-\cos(x_1)}{2}$，$\mu_{M_2}(x_1) = 1-\mu_{M1}(x_1)$，及 $\tau = 1, \sigma = 0, \rho = 1.2, \lambda_1 = 1, \lambda_2 = 1.2, \lambda_3 = 6, \lambda_4 = 5; \varepsilon_1 = \varepsilon_2 = 0.11, \varepsilon_3 = 0.2, \varepsilon_4 = 3.1, \varepsilon_5 = 2.1$。根据定理 2，通过 Matlab 中 LMI 工具箱可求解控制器为：$K_1 = [-1.4275 \quad -2.0628]$，$K_2 = [1.1392 \quad -2.0171]$。

选取初始值为 $[-0.9 \quad 0.7]$，$w(t) = 0.5e^{-2t}$，利用 MATLAB 仿真，图 1 是系统的状态响应，图 2 是控制律。由仿真结果可以看出，闭环系统是在 H_∞ 性能指标 ρ 下鲁棒渐近稳定。

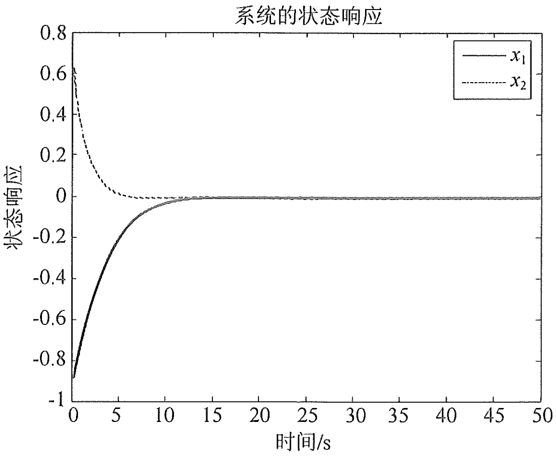

图 1　系统状态响应曲线

Fig. 1　State trajectory of system

4　结　论

本文研究了一类带有时变时滞的连续不确定模糊系统的时滞相关鲁棒 H_∞ 控制问题。在推导过程中，没有采用模型转换和边界不等式，而是引入了多个包含时滞项的模糊自由权值矩阵。基于模糊 LKF 和并行分布补偿算法，得到了闭环系统时滞相关鲁棒稳定新的条件，且控制器可以通过一组 LMIs 的解求得。最后由算例验证了该方法的有效性。

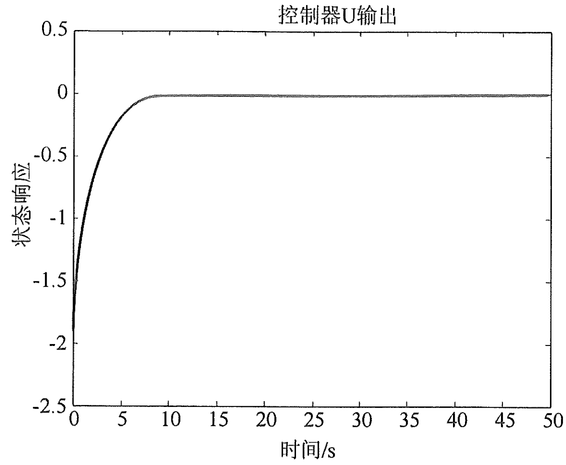

图 2　控制曲线

Fig. 2　Control trajectory

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